

應用數學 試題

1. Use a partial fractions to find an antiderivative of $f(x) = \frac{x+2}{x^3+x}$ (10%)

2. Please solve $\int \sin^4 x \cdot \cos^3 x dx$ (15%)

3. Please solve the following initial value problem

$$(xe^x + 2)y' + (1 + e^x y + xe^x y) = 0, y(0) = 1 \quad (20\%)$$

4. Find a formal Fourier series solution of the following problem:

$$2x'' + 10x = 10t, \text{ for } 0 < t < 1, \text{ with } x'(0) = x'(1) = 0 \quad (20\%)$$

5. Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$,

(1) Find the eigenvalues and corresponding eigenvectors. (10%)

(2) Find a matrix B such that $B^{-1}AB = D$ diagonal matrix, and write out D (10%)

6. Let $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ -1 & 2 & -3 & 4 \\ 3 & -3 & x^2 & x \end{array} \right]$ be the augmented matrix of a system of linear equations over $\mathbf{R}$. Determine

all values of x such that

(1) the system has no solution. (5%)

(2) the system has infinitely many solutions. (5%)

(3) the system has a unique solution. (5%)